Northern Beaches Secondary College

Manly Selective Campus

2014 HSC –Trial Examination

Mathematics Extension 2

General Instructions

- Reading time – 5 minutes.
- Working time – 3 hours .
- Write using blue or black pen.
- Board-approved calculators and templates may be used.
- All necessary working should be shown in every question.
- Multiple choice questions to be completed on the special answer page.
- Each free response questions to be completed in separate booklets.
- If using more than one booklet per question, number booklet "1 of __"

Total marks – 100 marks

- Attempt Questions 1-16
- Multiple Choice – answer question on answer sheet provided.
- Multiple Choice – 1 mark per question
- Short Answer questions – marks as indicated.

MULTIPLE CHOICE SECTION.

Answer the following questions on the answer sheet provided.

Q1. If $z = 1 + 2i$ and $\omega = 3 - i$ then $z - \overline{\omega}$ is:

$i - 2$

$4 + i$

$3i - 2$

$4 + 3i$

Q2. The directrices of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ are :

A) $x = \pm \frac{16\sqrt{7}}{7}$

B) $x = \pm \frac{\sqrt{7}}{16}$

C) $x = \pm \frac{16}{5}$

D) $x = \pm \frac{5}{16}$

Q3. Let α, β , γ be the roots of the equation $x^3 + x - 1 = 0$. The polynomial equation with roots 2αβ, 2αγ , 2βγ is:

A $x^3 + 2x^2 + 2 = 0$

B $x^3 - 2x^2 - 2 = 0$

C $x^3 - 2x^2 - 8 = 0$

D $x^3 - 2x^2 + 8 = 0$

Q4. If $f(x) = \dfrac{x(x-1)}{x-2}$, which of the following lines will be an asymptote to the graph $y = f(x)$?

A) $y = x + 1$

B) $y = x - 2$

C) $y = x - 1$

D) $y = 0$

Q5. P(z) is a polynomial of degree 4. Which one of the following statements must be false?

A P(z) has no real roots.

B *P(z)* has 1 real root and 3 non-real roots

C *P(z)* has 2 real roots and 2 non-real roots

D *P(z)* has 4 non-real roots

Q6. For a certain function $y = f(x)$, the function $= f(|x|)$ is represented by :

A) A reflection of $y = f(x)$ in the y –axis

B) A reflection of $y = f(x)$ in the x –axis.

C) A reflection of $y = f(x)$ in the x axis for $y \geq 0$

D) A reflection of $y = f(x)$ in the y axis for $x \geq 0$

Q7. If ω is an imaginary cube root of unity, then $\left(1 + \omega - \omega^2\right)^7$ is equal to :

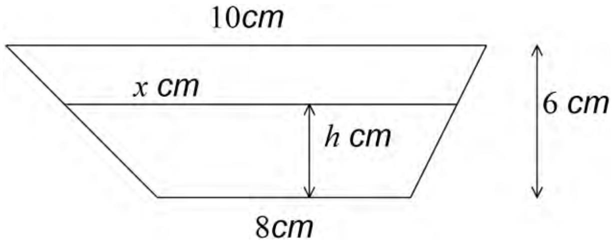
A) 128ω

B) -128ω

C) $128\omega^2$

D) $-128\omega^2$

Q8. The diagram shows a trapezium with an interval of x units drawn parallel to and h units above the base.



An expression for x in terms of h is given by:

A) $x = 5 - \frac{h}{6}$

B) $x = 8 + \frac{h}{6}$

C) $x = 12 + \frac{h}{8}$

D) $x = 8 + \frac{h}{3}$

Q9. Given the eccentricity of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is e, then the eccentricity of the ellipse $\frac{x^2}{a^2 + b^2} + \frac{y^2}{b^2} = 1$ is :

A) $\sqrt{e}$

B) $\frac{1}{e}$

C) e

D) e^2

Q10. The roots of the polynomial $4x^3 + 4x - 5 = 0$ and α, β, γ. The value of $(\alpha + \beta - 3\gamma)(b + \gamma - 3\alpha)(\alpha + \gamma - 3\beta)$ is

A) -80

B) -16

C) 16

D) 80

FREE RESPONSE SECTION – <u>answer each question in a separate booklet</u>

Question 11 START A NEW BOOKLET **15 marks**

a) (i) Find the Cartesian equation of the locus of of the point z in the complex plane given that $Re(z) = |z - 2|$ (2)

(ii) Sketch the locus of z. (1)

b) Let $P(z)$ be a point in the complex plane such that $|z - i| = \frac{1}{2}$. Find the maximum value of Arg (z) (2)

c) (i) Express $\sqrt{8 + 6i}$ in the form $x + iy$ where $x > 0$ (2)

(ii) Solve the equation $z^2 + 2(1 + 2i)z - (11 + 2i) = 0$ (2)

d) Let $f(x)$ be a continuous function for $-5 \leq x \leq 10$ and let $g(x) = f(x) + 2$.
If $\int_{-5}^{10} f(x)\, dx = 4$, show that $\int_{-5}^{10} g(u)\, du = 34$ (3)

e) Use integration by parts to find $\int_0^1 \frac{\cos^{-1} x}{\sqrt{1 + x}}\, dx$ (3)

Question 12 START A NEW BOOKLET **15 marks**

a) (i) Prove that any equation of the form

$$x^3 - mx^2 + n = 0$$

where m , $n \neq 0$ cannot have a triple zero. (2)

(ii) If the equation has a double zero, find the relation between m and n (2)

b) Use the table of standard integrals to find $\int \frac{2x}{\sqrt{x^4 + 16}}\, dx$ (2)

c) The equation $2x^2 - kx + 17 = 0$ has one zero, α, such that $Re(\alpha) = \frac{5}{2}$.

If k is real, find the other zero of the equation and the value of k. (2)

d) (i) Use the substitution $t = \tan\frac{x}{2}$ to prove

$$\int_0^{\frac{\pi}{2}} \frac{dx}{2 + 2\sin x} = \frac{1}{2}$$ (3)

(ii) Use the substitution $u = 2a - x$ to prove

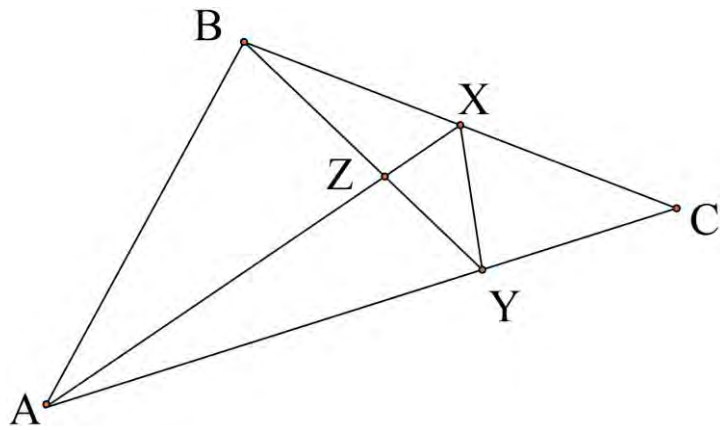
$$\int_0^{2a} f(x)\, dx = \int_0^{a} \{f(x) + f(2a - x)\}\, dx$$ (2)

(iii) Hence, or otherwise , evaluate

$$\int_0^{\pi} \frac{x}{2 + 2\sin x}\, dx$$ (2)

Question 13 START A NEW BOOKLET **15 marks**

a)



X and Y are points on sides BC and AC of $\Delta\, ABC$.

$\angle AXC = \angle BYC$ and $BX = XY$

(i) Copy the diagram into your answer booklet, clearly showing the above information. (Your diagram must be at least ¼ page in size) (1)

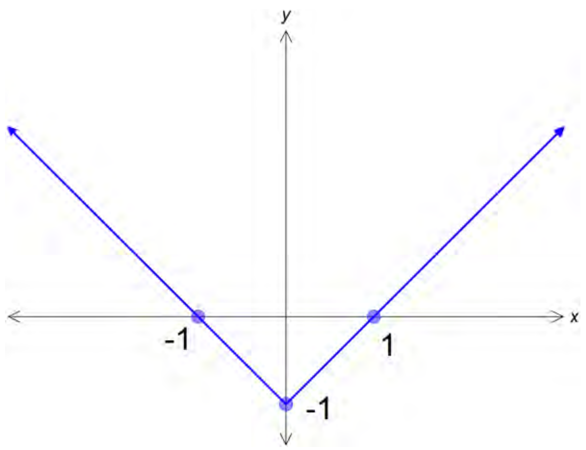
(ii) Prove $ABXY$ is a cyclic quadrilateral. (2)

(iii) Hence, or otherwise, prove AX bisects $\angle BAC$. (3)

Question 13 continued on next page

Question 13 continueda

b) Consider the function $f(x) = |x| - 1$ shown below.



Using diagrams, of at least ¼ page, sketch the following functions.
Show all essential features on your diagram.

(i) $y = 1 - f(x)$ (1)

(ii) $y = x.f(x)$ (2)

(iii) $|y| = f(x)$ (2)

(iv) $y = e^{f(x)}$ (2)

(v) $y = \sqrt{f(x)}$ (2)

Question 14 START A NEW BOOKLET **15 marks**

a) (i) Show that $(2 + i)$ is a root of $x^3 - 11x + 20 = 0$ (1)

(ii) Hence, or otherwise solve $x^3 - 11x + 20 = 0$ (2)

b) *A, B, C* and *D* are the vertices, in clockwise order, of a square.

(i) Given that *A* and *C* represent the points $2 + 2i$ and $4 + 2i$ respectively, find the coordinates of *B* and *D*. (2)

(ii) If the square *ABCD* is rotated anticlockwise through 90° about the origin, find the coordinates of the new position of the point *A*. (1)

c) Consider chords of the hyperbola $xy = c^2$ which pass through the point $A\,(6c\,,\,4c)$

(i) Find the equation of the chord passing through the points $P\left(cp,\ \frac{c}{p}\right)$ and $Q\left(cq,\ \frac{c}{q}\right)$. Express your answer in general form. (2)

(ii) Show that $4pq + 6 = p + q$ (1)

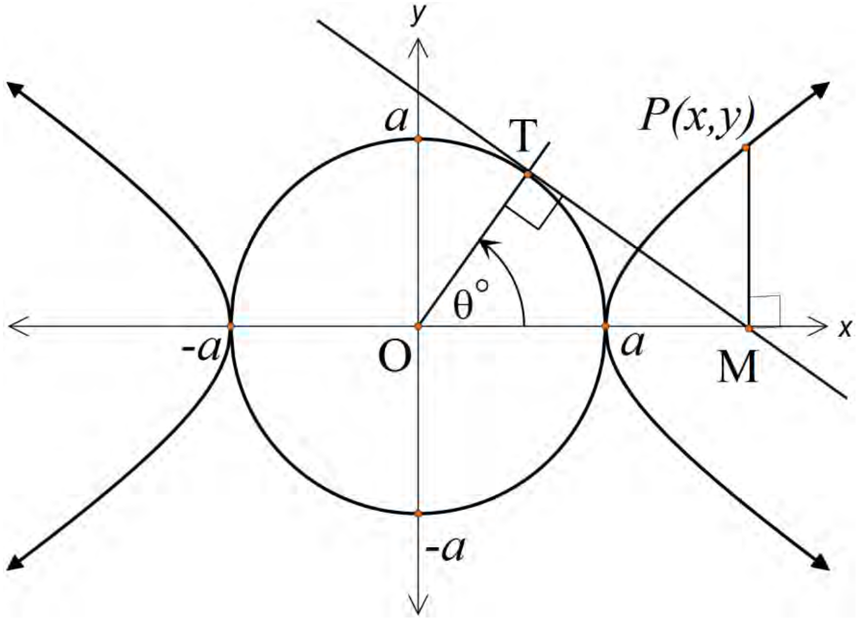
(iii) Find the equation of the locus of the midpoints of the chords *PQ*. (2)

d) (i) Find $\frac{dy}{dx}$ for the implicit function $x^2 - y^2 + xy + 5 = 0$. (2)

(ii) Find the *x* coordinate of the points of the curve $x^2 - y^2 + xy + 5 = 0$ where the tangents have a zero gradient. (2)

Question 15 START A NEW BOOKLET **15 marks**

a) The diagram below shows the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ and the circle $C: x^2 + y^2 = a^2$



The point P(x,y) lies on the hyperbola H and the point T lies on the circle C and $\angle TOM = \theta$.

(i) Show that P has the coordinates $(a\sec\theta,\ b\tan\theta)$. (2)

(ii) The point Q lies on the hyperbola and has coordinates $(a\sec\phi,\ b\tan\phi)$.

Given that $\theta + \phi = \frac{\pi}{2}$ and $\theta \neq \frac{\pi}{4}$, show that the chord PQ has the equation

$ay = b(\cos\theta + \sin\theta)x - ab$ (4)

(iii) Show that every chord PQ passes through a fixed point and find the coordinates of that point. (2)

(iv) Show that, as Θ approaches $\frac{\pi}{2}$, the chord PQ approaches a line parallel to an asymptote. (2)

Question 15 continued on next page

Question 15 continued

b) Let x_1 , x_2 be positive real numbers.

(i) Show that $x_1 + x_2 \geq 2\sqrt{x_1 x_2}$ (1)

(ii) Hence, or otherwise, prove

$$\frac{x_1 + x_2 + x_3 + x_4}{4} > \sqrt[4]{x_1 x_2 x_3 x_4}$$

where x_1, x_2, x_3, x_4 are positive real numbers. (2)

(iii) By making the substitution $x_4 = \frac{x_1 + x_2 + x_3}{3}$ into the result in part (ii) , show that

$$\frac{x_1 + x_2 + x_3}{3} > \sqrt[3]{x_1 x_2 x_3}$$

where x_1, x_2, x_3 are positive real numbers. (2)

Question 16 START A NEW BOOKLET **15 marks**

a) A solid has its base in the ellipse $E: \frac{x^2}{36} + \frac{y^2}{16} = 1$

If each section perpendicular to the major axis is an equilateral triangle, show that the volume of the solid is $128\sqrt{3}$ cubic units. (4)

b) Let $I_n = \int_0^1 x\left(1 - x^5\right)^n dx$ where $n \geq 0$ is an integer

(i) Show that $I_n = \frac{5n}{5n+2} I_{n-1}$ for $n \geq 1$ (3)

(ii) Show that $I_n = \frac{5^n\, n!}{2 \times 7 \times 12 \times \ldots \times (5n+2)}$ for $n \geq 1$ (2)

(iii) Evaluate I_4 (1)

c) Let *f(x), g(x)* be continuously differentiable functions.

(i) Prove $\int \{f''(x)\, g(x) - f(x)\, g''(x)\}\, dx = f'(x)\, g(x) - f(x)\, g'(x)$ (2)

(ii) Hence, or otherwise, evaluate $\int_0^{\frac{\pi}{4}} \sin 3x \cos 2x \, dx$ (3)

(State your answer in simplest exact form)

End of Examination

Northern Beaches Secondary College
Manly Selective Campus

2014 HSC Mathematics Extension 2
Trial Examination

Answer sheet for multiple choice questions 1- 10

INSTRUCTIONS:

For Questions 1 to 10, place a cross in the box corresponding to your selected answer.

Student Number: ___________________________________

Question #	*(A)*	*(B)*	*(C)*	*(D)*
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

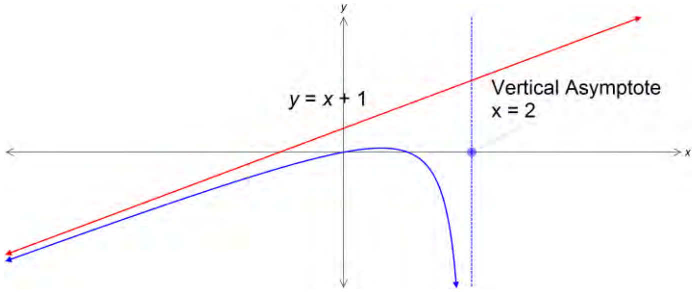
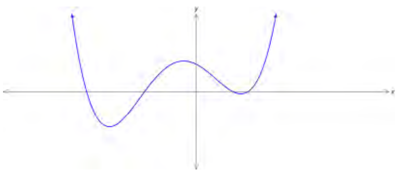
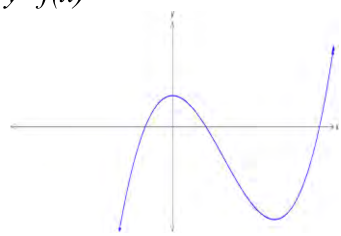
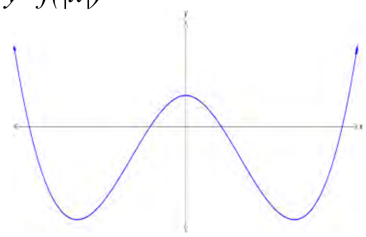
$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

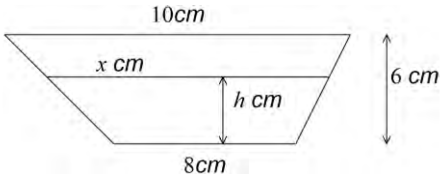
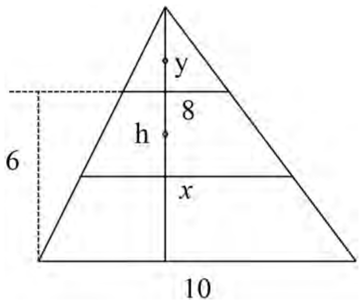
$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE : $\ln x = \log_e x, \quad x > 0$

Question 1:

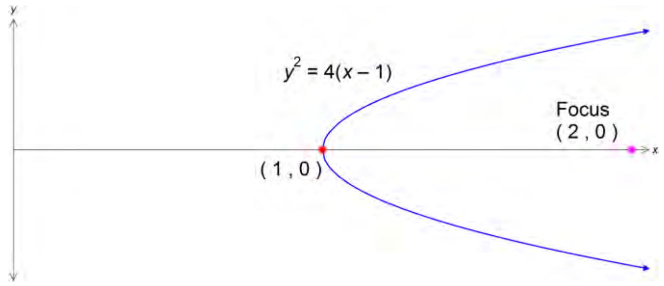
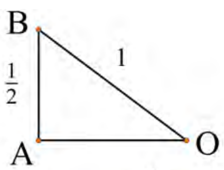
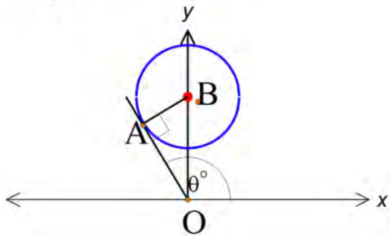
Q1	$z = 1 + 2i \quad \omega = 3 - i \therefore \overline{\omega} = 3 + i$ $z - \overline{\omega} = 1 + 2i - (3 + i)$ $= i - 2$	***A***
Q2	$\frac{x^2}{16} + \frac{y^2}{9} = 1$ Directrices: $x = \pm\frac{a}{e} = \pm\frac{4}{e} \qquad e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{\sqrt{7}}{4}$ $x = \pm\frac{16}{\sqrt{7}} = \pm\frac{16\sqrt{7}}{7}$	***A***
Q3	$x^3 + x - 1 = 0$ $\therefore \quad \alpha\beta\gamma = 1$ $2\alpha\beta = \frac{2\alpha\beta\gamma}{\gamma} = \frac{2}{\gamma}$ $2\alpha\gamma = \frac{2}{\beta}$ $2\beta\gamma = \frac{2}{\alpha}$ $\therefore \quad x = \frac{2}{X}$ $\frac{8}{X^3} + \frac{2}{X} - 1 = 0$ $8 + 2X^2 - X^3 = 0$ $\therefore \quad x^3 - 2x^2 - 8 = 0$	***C***

Q4	$f(x) = \dfrac{x(x-1)}{x-2}$ 	***A***
Q5	It is not possible to have one real root and 3 unreal roots – see diagram. 	***B***
Q6	Resulting graph is reflection of positive *x* values in the *y* axis. *y=f(x)*  *y=f(|x|)* 	***D***
Q7	$\omega^3 - 1 = 0$ $\therefore (\omega - 1)(\omega^2 + \omega + 1) = 0 \quad \therefore (1 + \omega + \omega^2) = 0$ $(1 + \omega - \omega^2)^7 = (1 + \omega + \omega^2 - \omega^2 - \omega^2)^7$ $= (-2\omega^2)^7$ $= -128\omega^{14} = -128(\omega^{12}\omega^2)$ $= -128\omega^2$	***D***

Q8	 Short Method. – Linear method x is a function of h *ie.* $x = ah + b$ $h = 0\ ,\ x = 8$ $8 = 0 + b \Rightarrow b = 8$ $h = 6\ ,\ x = 10$ $10 = 6a + 8$ $a = \frac{1}{3}$ $\therefore\ x = \frac{h}{3} + 8$ Longer (tedious) method  $\frac{y}{8} = \frac{y+6}{10} \Rightarrow y = 24$ $\frac{y}{8} = \frac{y+h}{x}$ $\frac{24}{8} = \frac{24+h}{x}$ $3x = 24 + h$ $x = 8 + \frac{h}{3}$	***D***

Q9	$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1 \qquad e^2=\left(\frac{b^2}{a^2}-1\right)$ $\therefore \qquad b^2=a^2(e-12)$ $\frac{x^2}{a^2+b^2}+\frac{y^2}{b^2}=1 \ \therefore E^2=1-\frac{b^2}{a^2+b^2}$ $\therefore \qquad E^2=\frac{a^2+b^2-b^2}{a^2+b^2}=\frac{a^2}{\left(a^2+b^2\right)}$ $=\frac{a^2}{a^2+a^2\left(e^2-1\right)}=\frac{1}{e^2}$ $\therefore \qquad E=\pm\frac{1}{e}$	***B***
Q10	$4x^3+4x-5=0$ $\alpha+\beta+\gamma=0 \quad \alpha\beta\gamma=\frac{5}{4}$ $(\alpha+\beta-3\gamma)(b+\gamma-3\alpha)(\alpha+\gamma-3\beta)$ $=(\alpha+\beta+\gamma-4\gamma)(\alpha+\beta+\gamma-4\alpha)(\alpha+\beta+\gamma-4\beta)$ $=-64\alpha\beta\gamma=-\frac{320}{4}=-80$	***A***

Question 11

a-i	$Let\ z = x + iy \rightarrow Re(z) = x$ $Re(z) = \|z - 2\|$ $\therefore\ x = \|z - 2\|$ $x^2 = (x - 2)^2 + y^2$ $x^2 = x^2 - 4x + 4 + y^2$ $y^2 = 4(x - 1)$	***2 marks – correct solution*** $x = \|z - 2\|$ ***1 mark – recognising***
a-ii		***1 mark correct solution with (1,0)***
b	$\|z - i\| = \frac{1}{2}$ $x^2 + (y - 1)^2 = \frac{1}{4}$ $\therefore\ \theta = \frac{\pi}{2} + \frac{\pi}{6} = \frac{2\pi}{3}$	***2 marks – correct solution fully demonstrated.*** ***1 mark – diagram showing circle and triangle correctly***

c-i	$\sqrt{8+6i} = x + iy$ $\therefore \quad x^2 - y^2 = 8$ $2xy = 6 \Rightarrow y = \dfrac{3}{x}$ $x^2 + \dfrac{9}{x^2} = 8$ $x^4 + 9 = 8x^2$ $x^4 - 8x^2 + 9 = 0$ $(x^2 - 9)(x^2 + 1) = 0$ $\therefore \quad x = \pm 3 \qquad nb\ x\ is\ real\ and\ \ x > 0$ $x = 3 \therefore y = 1$ $\sqrt{8+6\text{i}} = 3 + \text{i}$	***2 marks correct solution*** ***1 marks*** ***– correct except failure to recognise x > 0***
c-ii	$0 = z^2 + 2(1+2i)z - (11+2i)$ $z = \dfrac{-2(1+2i) \pm \sqrt{4(1+2i)^2 + 4 \times (11+2i)}}{2}$ $= \dfrac{-2(1+2\text{i}) \pm 2\sqrt{1 - 4 + 4\text{i} + 11 + 2\text{i}}}{2}$ $= \left(-(1+2\text{i}) \pm \sqrt{8+6\text{i}} \right)$ $= -1 - 2\text{i} \pm (3 + \text{i})$ $z = -1 - 2i + 3 + i = 2 - i\ or\ z = -1 - 2i - 3 - i = -4 - 3i$	***2 marks – correct solution*** ***1 mark – uses quadratic formula to arrive at non-simplified expression for z***

d	$\int_{-5}^{10} f(x)\,dx = 4$ $\therefore \int_{-5}^{10} g(x)\,dx = \int_{-5}^{10} f(x) + 2\,dx = 4 + \left[2x\right]_{-5}^{10}$ $= 4 + 30 = 34$ *if* $u = x$ *then* $du = dx$ $nb \int g(x)dx = \int g(u)\,du = 34$	***3 marks – correct solution – MUST include logic for*** $\int g(x)dx = \int g(u)\,du$ ***2 marks – correct except for explanation.*** ***1 mark – expansion of g(x) integral***
e	$\int_0^1 \frac{\cos^{-1}x}{\sqrt{1+x}}\,dx$ $u = \cos^{-1}x \qquad v' = \frac{1}{\sqrt{1+x}}$ $u' = -\frac{1}{\sqrt{1-x^2}} \qquad v = \frac{\sqrt{1+x}}{\frac{1}{2}}$ $I = \left[\cos^{-1}x \times 2\left(\sqrt{1+x}\right)\right]_0^1 + 2\int_0^1 \frac{\sqrt{1+x}}{\sqrt{1+x}\,\sqrt{1-x}}dx$ $= \left(0 \times 2\sqrt{2} - 2 \times \frac{\pi}{2} \times 1\right) + 2\int_0^1 (1-x)^{-\frac{1}{2}}\,dx$ $= -\pi + 2 \times -2\left[\sqrt{1-x}\right]_0^1$ $= -\pi - 4(0-1)$ $= -\pi + 4$	***3 marks – correct solution*** ***2 marks – correct value for initial term ie. -π*** ***1 mark – correct initial u, u' ,v and v'***

Question 12

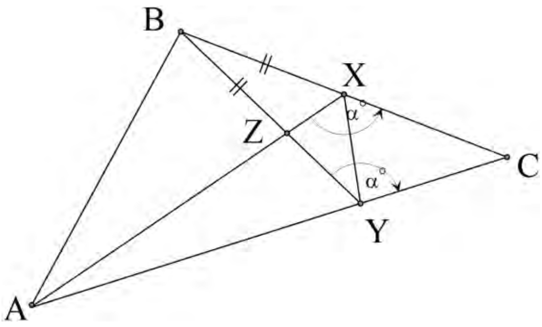
a	$f(x) = x^3 - mx^2 + n$ $f'(x) = 3x^2 - 2mx$ $f''(x) = 6x - 2m = 0$ $\therefore \quad x = \frac{m}{3}$ $if\ f'\left(\frac{m}{3}\right) = \frac{3m^2}{9} - \frac{2m^2}{3}$ $= -\frac{m^2}{3} \neq 0\ if\ m \neq 0$ therefore not possible to have triple root.	*2 marks for correct solution* *1 mark for* $x = \frac{m}{3}$
b	$f(x) = x^3 - mx^2 + n$ $f'(x) = 3x^2 - 2mx$ $\therefore \quad x = 0\ or\ x = \frac{2m}{3}$ $x = 0$ *not a solution for* $f(x)$ $f\left(\frac{2m}{3}\right) = \frac{8m^3}{27} - \frac{4m^3}{9} + n$ $= -\frac{4m^3}{27} + n = 0$ $4m^3 = 27n$	*2 marks for correct solution* *1 mark for* $x = \frac{2m}{3}$
c	$\int \frac{2x}{\sqrt{x^4 + 16}}\, dx$ $let\ u = x^2 \therefore du = 2xdx$ $= \int \frac{1}{\sqrt{u^2 + 4^2}} du$ $= \ln\left(u + \sqrt{u^2 + 16}\right) + C$ $= \ln\left(x^2 + \sqrt{x^4 + 16}\right) + C$	*2 marks for correct solution* *1 mark for* $\int \frac{du}{\sqrt{u^2 + 1}}$

d-i	$2x^2 - kx + 17 = 0$ $\alpha + \beta = \frac{k}{2}$ $\alpha = \frac{5}{2} + ib$ $\beta = \frac{5}{2} - ib$ $\therefore \quad \alpha + \beta = 5$ $\therefore \quad k = 10$ $x = \frac{k \pm \sqrt{k^2 - 136}}{4}$ $x = \frac{10 \pm \sqrt{100 - 136}}{4}$ $= \frac{5}{2} \pm \frac{3}{2}\,i$	***2 marks for correct solution***
d-ii	$\int_0^{\frac{\pi}{2}} \frac{dx}{2 + 2\sin x} = \frac{1}{2}$ $t = \tan\frac{x}{2} \;\therefore\; x = 2\tan^{-1} t$ $dx = \frac{2}{1 + t^2}dt \quad also\ \sin x = \frac{2t}{1 + t^2}$ $x = 0 \quad t = 0$ $x = \frac{\pi}{2} \quad t = 1$ $\int_0^1 \frac{dt}{2 + 2 \times \frac{2t}{1 + t^2}} \times \frac{2}{1 + t^2}$ $= \int_0^1 \frac{dt}{t^2 + 2t + 1}$ $= \int_0^1 \frac{dt}{(t + 1)^2}$ $= \quad -1\left[\frac{1}{t + 1}\right]_0^1 = 1 - \frac{1}{2} = \frac{1}{2}$	***2 marks for correct expression for transform. integral*** ***1 mark for correct simplified expression for*** $\frac{1}{2 + 2\sin x}$ ***in terms of t***

	Using $u = 2a - x \therefore -du = dx$ $x = 0 \quad u = 2a$ $x = a \quad u = a$ $-\int_{2a}^{a} f(u)\,du = \int_{a}^{2a} f(u).\,du$ $nb \int_{a}^{2a} f(u)\,du = \int_{a}^{2a} f(x)\,dx$ $\therefore \int_{0}^{a} f(x)dx + \int_{a}^{2a} f(x)\,dx = \int_{0}^{2a} f(x)\,dx$ $\therefore \int_{0}^{2a} f(x)\,dx = \int_{0}^{a} \{f(x) + f(2a-x)\}\,dx$ *Alternatively* $\int_{0}^{a} \{f(x) + f(2a-x)\}\,dx = \left[F(x) - F(2a-x)\right]_{0}^{a}$ $= [F(a) - F(a)] - [F(0) - F(2a)]$ $= F(2a) - F(0)$ $\int_{0}^{2a} f(x)dx = \left[F(x)\right]_{0}^{2a} = F(2a) - F(0)$	***2 marks for correct solution*** ***1 mark for decomposition of integral in terms of x and u.***

	$\displaystyle\int_0^{\pi} \frac{x}{2+2\sin x}\,dx = \int_0^{\frac{\pi}{2}} \frac{x}{2+2\sin x} + \frac{\pi - x}{2+2\sin(\pi - x)}dx$ $\displaystyle = \int_0^{\frac{\pi}{2}} \frac{x}{2+2\sin x} + \frac{\pi - x}{2+2\sin(x)}dx$ $\displaystyle = \int_0^{\frac{\pi}{2}} \frac{\pi}{2+2\sin x}\,dx$ $\displaystyle = \pi\int_0^{\frac{\pi}{2}} \frac{dx}{2+2\sin x}$ $\displaystyle = \pi \times \frac{1}{2} = \frac{\pi}{2}$	***2 marks for correct solution*** ***1 mark for correct transformation of integral***

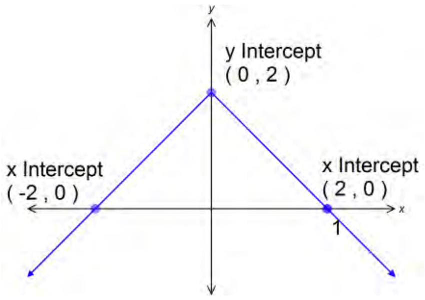
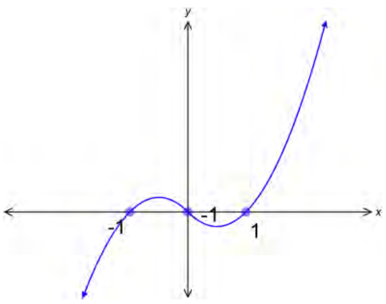
Question 13

a-i		***1 mark – correct diagram with all given details***
a-ii	*ABXY* is a cyclic quadrilateral. *Let* $\angle AXC = \angle BYC = \alpha$ *given* $\angle BXZ = 180^\circ - \alpha$ *Straight line* $\angle AYB = 180^\circ - \alpha$ *Straight line* $\therefore \angle BXZ = \angle AYB$ $\therefore$ *A, B, X, Y are concylic* (*angles standing on same arc are equal)* $\therefore$ *ABXY is a cyclic quad.*	***2 marks – correct solution including all supporting statements.*** ***1 mark – correct proof to two angles equal***

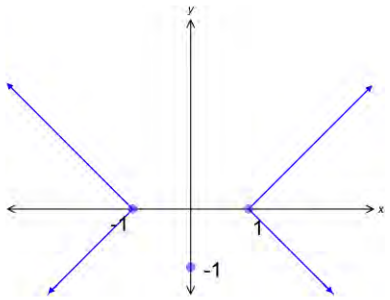
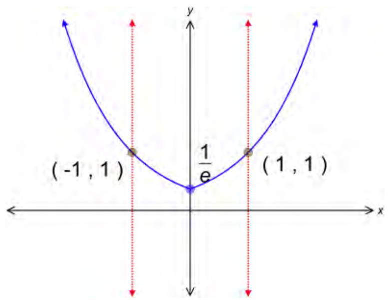
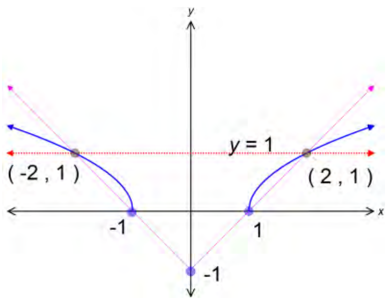
<table>
<tr><td>**a-iii**</td><td>prove AX bisects $\angle BAC$

$AX = XY$ *given*
$\therefore \angle XBY = \angle XYB$ *base $\angle$'s of isos. Δ*
$\angle XYB = \angle XAB$ *angles standing on same arc equal*
$\angle XBY = \angle XAY$ *angles standing on same arc equal*
$\therefore \angle XBY = \angle XAB$
$\therefore \angle XAY = \angle XAB$
$\therefore$ AX bisects $\angle BAY$</td><td>***3 marks – fully explained correct solution***

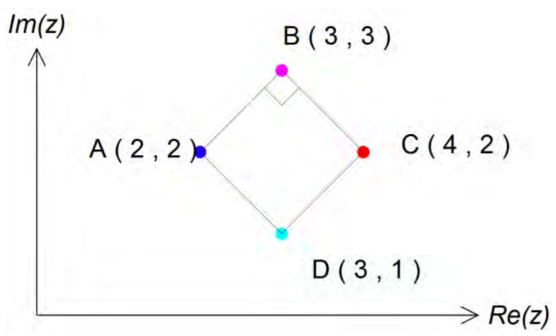
2 marks – correct logic but incomplete reasoning

1 mark – connection made between 3 equal angles</td></tr>
<tr><td>**b-i**</td><td>$y = 1 - f(x)$
</td><td>***1 mark – correct shape and points labelled***</td></tr>
<tr><td>**b-ii**</td><td>$y = x.f(x)$
</td><td>***2 marks – correct shape and points labelled***

1 mark – correct shape – no or incorrect corresponding points</td></tr>
</table>

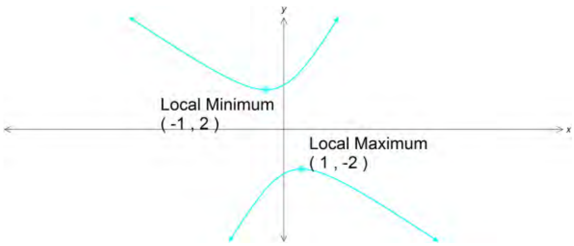
b-iii	$\|y\| = f(x)$ 	***2 marks – correct shape and points labelled*** ***1 mark – correct shape – no or incorrect corresponding points***
b-iv	$y = e^{f(x)}$ 	***2 marks – correct shape and points labelled*** ***1 mark – correct shape – no or incorrect corresponding points***
b-v	$y = \sqrt{f(x)}$ 	***2 marks – correct shape and points labelled and shape above and below y=1*** ***1 mark – correct shape – no or incorrect corresponding points***

Q14

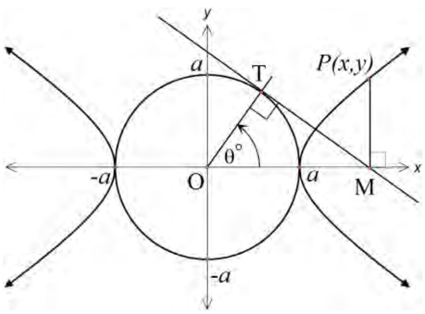
a-i	$x^3 - 11x + 20 = 0$ $x = 2 + i$ $\therefore \quad (2 + \text{i})^3 - 11(2 + \text{i}) + 20 = 0$ $8 + 12\text{i} + 6\text{i}^2 + \text{i}^3 - 22 - 11\text{i} + 20 = 0$ $(8 - 6 - 22 + 20) + (12\text{i} - \text{i} - 11\text{i}) = 0$ $0 = 0$ therefore (z + *i*) is a root of the equation.	***3 marks for correct solution*** ***2 marks for*** $P(2 + \text{i}) = 0$ $\therefore P(2 - \text{i}) = 0$ ***1 mark for*** *P(2 + i)=0*
a-ii	Since all coefficients are real then the conjugate is also a root ie; (z – *i*). therefore $x^2 - 2Re(x) + \|x\|^2$ is a factor. $\therefore x^2 - 4x + 2^2 + 1^2 = x^2 - 4x + 5$ *is s factor* $x^3 - 11x + 20 = (x^2 - 4x + 5)(x + \alpha)$ $\therefore \quad 20 = 5 \times \alpha$ $\alpha = 4$ $\therefore \quad (x^3 - 11x + 20) = (x^2 - 4x + 5)(x + 4)$ $x = 2 + 1 \quad x = 2 - i \quad x = -4$	
b-i	Find the coordinates of *B* (3 + 3i) and D (3 + i)  	***2 marks – correct solution*** ***1 mark – correct B or C***
b-ii	$A = 2 + 2\text{i}$ $\text{i}A = \text{i}(2 + 2\text{i}) = -2 + 2\text{i}$	***1 mark – correct answer.***

c-i	$m = \dfrac{\frac{c}{p} - \frac{c}{q}}{cp - cq} = \dfrac{q-p}{pq} \times \dfrac{1}{p-q} = -\dfrac{1}{pq}$ $y - \dfrac{c}{p} = -\dfrac{1}{pq}(x - cp)$ $ypq - cq = -x + cp$ $x + ypq = (p+q)c$ *Given the chord passes through A* $(6c, 4c)$	***2 marks for correct solution*** ***1 mark for correct gradient***
c-ii	*Given the chord passes through A* $(6c, 4c)$ $x + ypq = (p+q)c$ $6c + 4cpq = (p+q)c$ $6 + 4pq = (p+q)$	***1 mark – correct solution***
c-iii	$x = \dfrac{c(p+q)}{2} \qquad y = \dfrac{c\,(p+q)}{2pq} = \dfrac{x}{pq}$ $\therefore \qquad \dfrac{x}{y} = pq$ *also* $4pq + 6 = p + q$ $y = \dfrac{c(4pq+6)}{2pq}$ $ypq = \dfrac{c}{2}(4pq + 6)$ $y \times \dfrac{x}{y} = \dfrac{c}{2}\left(\dfrac{4x}{y} + 6\right)$ $xy = 2cx + 3cy$ $(x - 3c)y = 2cx$ $y = \dfrac{2cx}{x - 3c}$	***2 marks for correct solution*** ***1 mark for correct x or y coordinate of midpoint.***

d-i	$x = \frac{c(p+q)}{2} \qquad y = \frac{c\,(p+q)}{2pq} = \frac{x}{pq}$ $\therefore \qquad \frac{x}{y} = pq$ $also\; 4pq + 6 = p + q$ $x = \frac{c}{2}(4pq + 6)$ $x = \frac{2cx}{y} + 3c$ $x - 3c = \frac{2cx}{y}$ $y = \frac{2cx}{x - 3c}$ *CHECK* $ypq = \frac{c}{2}(4pq + 6)$ $y \times \frac{x}{y} = \frac{c}{2}\left(\frac{4x}{y} + 6\right)$ $xy = 2cx + 3cy$ $(x - 3c)y = 2cx$ $y = \frac{2cx}{x - 3c}$	

d-ii	$x^2 - y^2 + xy + 5 = 0$ $2x - \frac{2ydy}{dx} + y + \frac{xdy}{dx} = 0$ $(x - 2y)\frac{dy}{dx} = -(2x + y)$ $\frac{dy}{dx} = -\frac{2x + y}{x - 2y}$	***2 marks for correct solution*** ***1 mark for correct derivative expression***
	Zero gradient at $-2x = y$ $x^2 - y^2 + xy + 5 = 0$ $x^2 - 4x^2 - 2x^2 + 5 = 0$ $-5x^2 + 5 = 0$ $x = \pm 1$ $\therefore \quad -2x = y$ $x = 1 \quad y = -2$ $x = -1 \quad y = 2$  	***2 marks for correct solution*** ***1 mark ofr correct equation from*** $\frac{dy}{dx} = 0$

Question 15

	 $OT = a$ $\sec\theta = \dfrac{OM}{a} \quad \Rightarrow \; OM = a\sec\theta$ $H: \dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ $\therefore \dfrac{a^2\sec^2\theta}{a^2} - \dfrac{y^2}{b^2} = 1$ $\sec^2\theta - 1 = \dfrac{y^2}{b^2}$ $\tan^2\theta = \dfrac{y^2}{b^2}$ $\therefore \quad y = b\tan\theta$ (*first quadrant*)	***2 marks – fully demonstrated.*** ***1 mark – demonstrated by substitution into equation.***

<table>
<tr>
<td></td>
<td>

Given that $\theta + \phi = \frac{\pi}{2}$ and $\theta \neq \frac{\pi}{2}$, show that the chord PQ has the equation

$$ay = b(\cos\theta + \sin\theta)x - ab$$

$$m = \frac{b(\tan\theta - \tan\phi)}{a(\sec\theta - \sec\phi)}$$

$$= \frac{b}{a}\left\{\frac{\sin\theta}{\cos\theta} - \frac{\sin\phi}{\cos\phi}\right\} \div \left\{\frac{1}{\cos\theta} - \frac{1}{\cos\phi}\right\}$$

$$= \frac{b}{a}\left\{\frac{\sin\theta\cos\phi - \sin\phi\cos\theta}{\cos\theta\cos\phi}\right\} \times \left\{\frac{\cos\theta\cos\phi}{\cos\phi - \cos\theta}\right\}$$

$$= \frac{b}{a}\frac{\sin(\theta - \phi)}{\cos\phi - \cos\theta}$$

$$nb\ \phi = \frac{\pi}{2} - \theta \qquad \therefore \qquad \cos\phi = \sin\theta$$

$$= \frac{b}{a}\frac{\sin\left(\theta - \left(\frac{\pi}{2} - \theta\right)\right)}{\sin\theta - \cos\theta} = \frac{b\sin\left(-\left(\frac{\pi}{2} - 2\theta\right)\right)}{a(\sin\theta - \cos\theta)}$$

$$= \frac{b}{a}\frac{(-\cos 2\theta)}{(\sin\theta - \cos\theta)}$$

$$= \frac{-b(\cos^2\theta - \sin^2\theta)}{a(\sin\theta - \sin\theta)}$$

$$= \frac{-b(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{a(\sin\theta - \cos\theta)}$$

$$= \frac{b(\cos\theta + \sin\theta)}{a}$$

$$y - b\tan\theta = \frac{b(\cos\theta + \sin\theta)}{a}(x - a\sec\theta)$$

$$ay - ab\tan\theta = b(\cos\theta + \sin\theta)x - ab - ab\tan\theta$$

$$ay = b(\cos\theta + \sin\theta)x - ab$$

</td>
<td>

4 marks – correct solution fully detailed.

3 marks – gradient equal to

$$m = \frac{b(\cos\theta + \sin\theta)}{a}$$

2 marks – some further simplification of gradient using complementary angles

1 mark

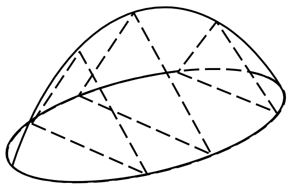
– recognition and some use of complementary angles

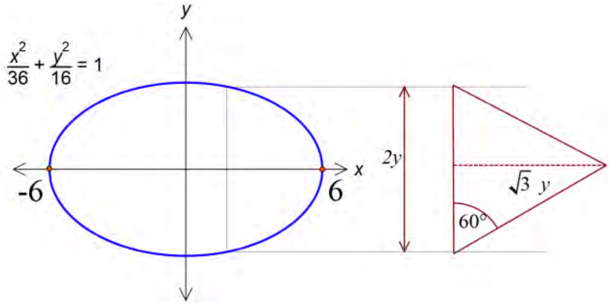
- initial partially simplified expression for gradient in terms of θ and ϕ

</td>
</tr>
</table>

	$ay = b(\cos\theta + \sin\theta)x - ab$ $x = 0 \qquad y = -b$ Therefore for all chords PQ $x = 0$, y will equal $-b$ irrespective of angle Θ ie. all chords will pass through the fixed point (0 , $-b$) since b is a constant.	***2 marks – correct solution fully explained.*** ***1 mark – partial explanation.***
	$ay = b(\cos\theta + \sin\theta)x - ab$ $asymptote\ y = \pm\frac{b}{a}x$ $as\ \theta \to \frac{\pi}{2}$ $\cos\theta \to \cos\frac{\pi}{2} = 0$ $\sin\theta \to \sin\frac{\pi}{2} = 1$ $\therefore \quad ay = b(0 + 1)x - ab$ $y = \frac{b}{a}x - b$ ie. *parallel to asymptote*	***2 marks – correct solution – did require identification of equation to the asymptote*** $y = \pm\frac{b}{a}x$ ***1 mark – correct equation without explanation.***
	$x_1 + x_2 \geq 2\sqrt{x_1 x_2}$ $(x + y)^2 \geq 0 \qquad for\ all\ x, y$ $x^2 + 2xy + y^2 \geq 0$ $x^2 + y^2 \geq 2xy$ $Let\ x \to \sqrt{x} \qquad y \to \sqrt{y}$ $\therefore (\sqrt{x})^2 + (sty)^2 \geq 2\sqrt{x}\sqrt{y}$ $x + y \geq 2\sqrt{xy}$	***1 mark – a correct solution.***

	$\frac{x_1+x_2+x_3+x_4}{4} \ge \sqrt[4]{x_1 x_2 x_3 x_4}$ $x_1+x_2 \ge 2\sqrt{x_1 x_2}$ $x_3+x_4 \ge 2\sqrt{x_3 x_4}$ $\therefore (x_1+x_2)+(x_3+x_4) \ge 2\sqrt{x_1 x_2}+2\sqrt{x_3 x_4}$ $\quad (a)$ $\frac{(x_1+x_2)+(x_3+x_4)}{2} \ge \sqrt{x_1 x_2}+\sqrt{x_3 x_4}$ $\sqrt{x_1 x_2}+\sqrt{x_3 x_4} \ge 2\sqrt{\sqrt{x_1 x_2}\sqrt{x_3 x_4}}$ $\therefore \frac{(x_1+x_2)+(x_3+x_4)}{2} \ge 2\times\sqrt[4]{x_1 x_2 x_3 x_4}$ $\frac{(x_1+x_2)+(x_3+x_4)}{4} \ge \sqrt[4]{x_1 x_2 x_3 x_4}$	***2 marks – correct solution*** ***1 mark – correct to line (a)***
	$\frac{x_1+x_2+x_3+x_4}{4} > \sqrt[4]{x_1 x_2 x_3 x_4}$ $let\ \frac{(x_1+x_2+x_3)}{3} = x_4$ $\frac{x_1+x_2+x_3}{4}+\frac{x_1+x_2+x_3}{12} > \sqrt[4]{\frac{x_1 x_2 x_3 (x_1+x_2+x_3)}{3}}$ $\frac{4(x_1+x_2+x_3)}{12} \ge \sqrt[4]{\frac{x_1 x_2 x_3 (x_1+x_2+x_3)}{3}}$ $\frac{x_1+x_2+x_3}{3} \ge (x_1 x_2 x_3)^{\frac{1}{4}} \times \left(\frac{x_1+x_2+x_3}{3}\right)^{\frac{1}{4}}$ $\quad (a)$ $\left(\frac{x_1+x_2+x_3}{3}\right)^{\frac{3}{4}} \ge (x_1 x_2 x_3)^{\frac{1}{4}}$ $\left(\frac{x_1+x_2+x_3}{3}\right)^{3} \ge (x_1 x_2 x_3)$ $\left(\frac{x_1+x_2+x_3}{3}\right) \ge \sqrt[3]{(x_1 x_2 x_3)}$	***2 marks – correct solution.*** ***1 mark – correct to line (a)***





	$$Area\ of\ Triangle = \frac{1}{2}bh = \frac{1}{2}2y\sqrt{3}\ y = \sqrt{3}\ y^2$$ $$\delta V = \sqrt{3}\ y^2 dx$$ $$V \approx 2\times \lim_{\delta x\to 0}\sum_{x=0}^{6}\sqrt{3}\ y^2\ \delta x$$ $$= 2\sqrt{3}\int_0^6 y^2\,dx$$ $$= 2\sqrt{3}\int_0^6 16\left(1-\frac{x^2}{36}\right)dx$$ $$= 32\sqrt{3}\times\int_0^6 1-\frac{x^2}{36}\,dx$$ $$= 32\sqrt{3}\left[x-\frac{x^3}{36}\right]_0^6$$ $$= 32\sqrt{3}\left(6-\frac{6^3}{3\times 36}\right)$$ $$= 32\sqrt{3}\times 4 = 128\sqrt{3}\ units^3$$	***4 marks for correct solution*** ***3 marks*** - ***Correct primitive function*** - ***Correct evaluation from incorrect*** δx ***2 marks for correct volume element and correct integral expression*** ***1 mark for correct area expression***

	$I_n = \int_0^1 x(1-x^5)^n\,dx$ $u = (1-x^5)^n \qquad v' = x$ $u' = -5nx^4(1-5x)^{n-1} \qquad v = \dfrac{x^2}{2}$ $I_n = \left[\dfrac{x^2}{2}\times(1-x^5)n\right]_0^1 - \int -\dfrac{5n}{2}x^4(1-5x)^{n-1}x^2\,dx$ $= 0 + \dfrac{5n}{2}\int_0^1 x^6(1-5x)^{n-1}\,dx$ $= \dfrac{5n}{2}\int_0^1 (-x)(-x^5)(1-5x)^{n-1}\,dx$ $= \dfrac{5n}{2}\int_0^1 (-x)\{(1-x^5)-1\}(1-x^5)^{n-1}\,dx$ $= \dfrac{5n}{2}\int_0^1 (-x)(1-x^5)^n - (-x)(1-x^5)^{n-1}dx$ $= -\dfrac{5n}{2}\int_0^1 (x)(1-x^5)^n - (x)(1-x^5)^{n-1}dx$ $I_n = -\dfrac{5n}{2}(I_n - I_{(n-1)})$ $I_n = -\dfrac{5n}{2}I_n + \dfrac{5n}{2}I_{n-1}$ $I_n + \dfrac{5n}{2}I_n = \dfrac{5n}{2}I_{n-1}$ $2I_n + 5n\,I_n = 5n\,I_{n-1}$ $(2+5n)I_n = 5n\,I_{n-1}$ $I_n = \dfrac{5n}{5n+2}I_{n-1}$	***3 marks for correct solution*** ***2 marks for correct decomposition of I_n using*** $x^6 = = x\{(1-x)^5 - 1\}$ ***1 mark for correct application fo IBP to obtain*** $\dfrac{5n}{2}\int_0^1 (1-5x)^{n-1}\,dx$

b-ii	$\int_0^1 (-x)(1-x^5)^n - (-x)(1-x^5)^{n-1} dx$ $I_n = \frac{5n}{5n+2} I_{n-1}$ $I_{n-1} = \frac{5(n-1)}{5(n-1)+2} I_{n-2} = \frac{5n-5}{5n-3} I_{n-2}$ $I_{n-2} = \frac{5(n-2)}{5(n-2)+2} I_{n-3} = \frac{5n-10}{5n-8} I_{n-4}$ $\therefore I_{n-4} = \frac{5n-15}{5n-13} I_{n-4} \dots$ $I_2 = \frac{5\times 2}{5\times 2+2} I_1$ $I_1 = \frac{5}{5+2} I_0$ $I_0 = \int_0^1 x(1-x)^0 \, dx = \left[\frac{x^2}{2}\right]_0^1 = \frac{1}{2}$ $\therefore$ $I_n = \frac{5n}{5n+2} \times \frac{5n+5}{5n-3} \times \frac{5n-10}{5n-8} \times \dots \times \frac{10}{12} \times \frac{5}{7} \times \frac{1}{2}$ $= \frac{5(n) \times 5(n-1) \times 5(n-2) \times \dots \times 5(2) \times 5(1) \times 1}{2 \times 7 \times 12 \times \dots \times (5n+2)}$ $= \frac{5^n \, n!}{2 \times 7 \times 12 \times \dots (5n+2)}$	***2 marks for correct solution*** ***1 mark for correct expression of initial three terms*** ***Or 1 mark for evaluation of I_0***
	$I_4 = \frac{5^4 \times 4!}{2 \times 7 \times 12 \times 17} = \frac{625}{119}$	***1 mark for correct answer***

<table>
<tr>
<td>c</td>
<td>

$\int \{f''(x)\, g(x) - f(x)\, g''(x)\}\, dx = f'(x)\, g(x) - f(x)\, g'(x)$

$\therefore \int \{f''(x)\, g(x) - f(x)\, g''(x)\}\, dx = \int f''(x)\, g(x) dx - \int f(x)\, g''(x)\, dx$

$\int f''(x)\, g(x) dx$

$u = g(x) \qquad v' = f''(x)$
$u' = g'(x) \qquad v = f'(x)$

$\therefore \int f''(x)\, g(x) dx = g(x) f'(x) - \int f'(x)\, g'(x) dx$ ①

$\int f(x)\, g''(x) dx$

$u = f(x) \qquad v' = g''(x)$
$u' = f'(x) \qquad v = g'(x)$

$\therefore \int f(x)\, g''(x) dx = g'(x) f(x) - \int f'(x)\, g'(x) dx$ ②

① – ②

$g(x) f'(x) - \int f'(x)\, g'(x) dx - \left\{ g'(x) f(x) - \int f'(x)\, g'(x) dx \right\}$

$= g(x) f'(x) - g'(x) f(x)$

$\therefore \int \{f''(x)\, g(x) - f(x)\, g''(x)\}\, dx = f'(x)\, g(x) - f(x)\, g'(x)$

</td>
<td>

2 marks for correct solution

1 mark for

- ***Correct application of product rule to RHS***
- ***Correct application of IBP to LHS***

</td>
</tr>
</table>

c-iii	$f(x) = \sin 3x \qquad g(x) = \cos 2x$ $f'(x) = 3\cos 3x \quad g'(x) \ -2\sin 2x$ $f''(x) = -9\sin 3x \qquad g''(x) = -4\cos 2x$ $\int f''(x)\, g(x) - f(x) g''(x)\, dx$ $= \int -9\sin 3x\cos 2x - \sin^3 x \times (-4\cos 2x)\, dx$ $= \int -5\sin 3x\cos 2x\, dx$ $= -5\int \sin 3x \cos 2x\, dx$ From the previous part (i) therefore $\int f''(x)\, g(x) - f(x)g''(x)\, dx = f'(x)g(x) - f(x)\, g'(x)$ $= -5\int \sin 3x \cos 2x\, dx$ $= -5\{3\cos 3x\cos 2x - \sin 3x \times (-2\sin 2x)\}$ $= -5\left[3\cos 3x\cos 2x + 2\sin 3x\sin 2x\right]_0^{\frac{\pi}{4}}$ $= -5\left\{\left(3 \times -\frac{1}{\sqrt{2}} \times 0 + 2 \times \frac{1}{\sqrt{2}} \times 1\right) - (3 \times 1 \times 1 + 2 \times 0 \times 0)\right\}$ $= -5\left(\frac{2}{\sqrt{2}} - 3\right)$ $\therefore \int \sin 3x \cos 2x = \frac{1}{5} \times \left(3 - \frac{2}{\sqrt{2}}\right)$	***3 marks for correct solution*** ***2 marks for substantial progress towards solution*** ***1 mark for relevant initial progress to a solution.***

	Or Method 2 $\int_0^{\frac{\pi}{4}} \sin 3x \cos 2x \, dx$ $\int \{f''(x)\, g(x) - f(x)\, g''(x)\}\, dx = f'(x)\, g(x) - f(x)\, g'(x)$ $u = \sin 3x \qquad v' = \cos 2x$ $u' = 3\cos 3x \qquad v = \frac{1}{2}\sin 2x$ $I = \frac{1}{2}\sin 3x \sin 2x - \frac{3}{2}\int \cos 3x \sin 2x$ $u = \cos 3x \qquad v' = \sin 2x$ $u' = \text{-}3\sin 3x \qquad v = -\frac{1}{2}\cos 2x$ $I_2 = -\frac{1}{2}\cos 3x \cos 2x - \frac{3}{2}\int \sin 3x \cos 2x \, dx$ $= -\frac{1}{2}\cos 3x \cos 2x - \frac{3}{2} I$ $I = \frac{1}{2}\sin 3x \sin 2x - \frac{3}{2}\left\{ -\frac{1}{2}\cos 3x \cos 2x - \frac{3}{2} I \right\}$ $= \frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x + \frac{9}{4} I$ $-\frac{5}{4} I = \left[\frac{1}{2}\sin 3x \sin 2x + \frac{3}{4}\cos 3x \cos 2x \right]_0^{\frac{\pi}{4}}$ $-5I = \left[2\sin 3x \sin 2x + 3\cos 3x \cos 2x \right]_0^{\frac{\pi}{4}}$ $\text{-}5I = \left(2 \times \frac{1}{\sqrt{2}} \times 1 + 3 \times -\frac{1}{\sqrt{2}} \times 0 \right) - (2 \times 0 \times 0 + 3 \times 1 \times 1)$ $I = \frac{-1}{5}\left(\frac{2}{\sqrt{2}} - 3 \right) = \frac{1}{5}\left(3 - \frac{2}{\sqrt{2}} \right)$	